Beyond the Standard Model Higgs Boson

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1 June 2018
MASS2018
CP$^3$ Origins,
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• Extended Higgs sector—motivations and constraints

• Evidence for a SM-like Higgs boson

• Beyond the SM Higgs boson—the Two Higgs Doublet Model (2HDM)

• The Higgs basis and the alignment limit

• The Higgs-fermion Yukawa couplings

• Avoiding flavor non-diagonal couplings of the Higgs bosons to fermions

• Achieving a SM-like Higgs boson in the 2HDM
  – Approximate alignment by accident
  – Approximate alignment via an approximate symmetry

• Conclusions
Why not an extended Higgs sector?

- The fermion and gauge boson sectors of the Standard Model (SM) are not of minimal form ("Who ordered that?"). So, why should the spin-0 (scalar) sector be minimal?

- Extended Higgs sectors can provide a dark matter candidate.

- Extended Higgs sectors can modify the electroweak phase transition and facilitate baryogenesis.

- Extended Higgs sectors can enhance vacuum stability.

- Models of new physics beyond the SM often require additional scalar Higgs states. E.g., two Higgs doublets are required in the minimal supersymmetric extension of the SM (MSSM).
Extended Higgs sectors are highly constrained

- The electroweak $\rho$ parameter is very close to 1.

- One neutral Higgs scalar of the extended Higgs sector must be SM-like (and identified with the Higgs boson at 125 GeV).

- Higgs-mediated flavor-changing neutral currents (FCNCs) are suppressed.

- Charged Higgs exchange at tree level (e.g. in $\bar{B} \rightarrow D^{(*)}\tau^-\bar{\nu}_\tau$) and at one-loop (e.g. in $b \rightarrow s\gamma$) can significantly constrain the charged Higgs mass and the Yukawa couplings.

- At present, only one Higgs scalar has been observed.

- If the scale that governs the non-SM-like Higgs bosons is close to the electroweak scale, is the naturalness problem of electroweak symmetry breaking exacerbated?
In addition, a number of other conditions are typically imposed that restrict the range of the scalar potential parameters.

- The Higgs scalar potential is bounded from below.

- The electroweak minimum conserves electric charge (and color).

- (Tree-level) unitarity and perturbativity of Higgs couplings.

- Consistency with precision electroweak constraints (primarily, the $S$, $T$ and $U$ parameters).

- Absence of Landau poles below the Planck scale.

- (Meta-)stability of the scalar potential (absence of a deeper minimum at field values below the Planck scale).
The $\rho$-parameter constraint on extended Higgs sectors

Given that the electroweak $\rho$-parameter is very close to 1, it follows that a Higgs multiplet of weak-isospin $T$ and hypercharge $Y$ must satisfy,\(^\text{1}\)

$$\rho \equiv \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1 \iff (2T + 1)^2 - 3Y^2 = 1,$$

independently of the Higgs vacuum expectation values (vevs). The simplest solutions are Higgs singlets $(T, Y) = (0, 0)$ and hypercharge-one complex Higgs doublets $(T, Y) = \left(\frac{1}{2}, 1\right)$. For example, the latter is employed by the two Higgs doublet model (2HDM).

More generally, one can achieve $\rho = 1$ by fine-tuning if

$$\sum_{T, Y} \left[4T(T + 1) - 3Y^2\right]|V_{T,Y}|^2 c_{T,Y} = 0,$$

where $V_{T,Y} \equiv \langle \Phi(T, Y) \rangle$ is the scalar vev, and $c_{T,Y} = 1$ for complex Higgs representations and $c_{T,Y} = \frac{1}{2}$ for real $Y = 0$ Higgs representations.

\(^{\text{1}}Y\) is normalized such that the electric charge of the scalar field is $Q = T_3 + Y/2$. 
Evidence for a SM-like Higgs boson

Best fit values of $\sigma_i \times \text{BR}^f$ for each specific channel $i \rightarrow H \rightarrow f$, for the combination of the ATLAS and CMS measurements. Taken from JHEP 08 (2016) 045.

Summary plot of best-fit values of the production-decay signal strength products $\mu^{if}_i = \mu_i \times \mu^f$ (the horizontal bars indicate the 1σ CL intervals). Taken from CMS-PAS-HIG-17-031.
• Higgs particle is still there! 😊
• In Run-2 the precision on e.g. cross sections/sensitivity will improve with a factor 2-3 wrt Run-1 results
• The mild deviations seen in Run-1 seem to be gone 😞
• Evidence for $H \rightarrow bb$ in the associated production channel
• Observation of $H \rightarrow \tau\tau$ in a single experiment
• $ttH$ is observed directly
• No deviations from Standard Model Higgs expectations yet!!

The Higgs Boson is still very much Standard Model-like!
The Two-Higgs doublet model (2HDM)

The 2HDM, consisting of two hypercharge-one, SU(2) doublet scalar fields \( \Phi_1 \) and \( \Phi_2 \), provides an instructive model for an extended Higgs sector. The 2HDM scalar potential is,

\[
V = m_{11}^2 \Phi_1^+ \Phi_1 + m_{22}^2 \Phi_2^+ \Phi_2 - [m_{12}^2 \Phi_1^+ \Phi_2 + \text{h.c.}] + \frac{1}{2} \lambda_1 (\Phi_1^+ \Phi_1)^2 \\
+ \frac{1}{2} \lambda_2 (\Phi_2^+ \Phi_2)^2 + \lambda_3 (\Phi_1^+ \Phi_1)(\Phi_2^+ \Phi_2) + \lambda_4 (\Phi_1^+ \Phi_2)(\Phi_2^+ \Phi_1) \\
+ \left\{ \frac{1}{2} \lambda_5 (\Phi_1^+ \Phi_2)^2 + [\lambda_6 (\Phi_1^+ \Phi_1) + \lambda_7 (\Phi_2^+ \Phi_2)] \Phi_1^+ \Phi_2 + \text{h.c.} \right\}.
\]

After minimizing the scalar potential, \( \langle \Phi_i^0 \rangle = v_i/\sqrt{2} \) (for \( i = 1, 2 \)) with \( v \equiv (|v_1|^2 + |v_2|^2)^{1/2} = 2m_W/g = 246 \) GeV.

Define the scalar doublet fields of the Higgs basis,

\[
H_1 = \left( \begin{array}{c} H_1^+ \\ H_1^0 \end{array} \right) \equiv \frac{v_1^* \Phi_1 + v_2^* \Phi_2}{v}, \hspace{1cm} H_2 = \left( \begin{array}{c} H_2^+ \\ H_2^0 \end{array} \right) \equiv \frac{-v_2 \Phi_1 + v_1 \Phi_2}{v},
\]

such that \( \langle H_1^0 \rangle = v/\sqrt{2} \) and \( \langle H_2^0 \rangle = 0 \).
The Goldstone boson fields are: $G^\pm = H_1^\pm$ and $G^0 = \sqrt{2} \text{Im} H_1^0$. The physical charged Higgs fields are $H^\pm = H_2^\pm$. The neutral Higgs mass eigenstate fields are linear combinations of $\sqrt{2} \text{Re} H_1^0 - v$, $\sqrt{2} \text{Re} H_2^0$ and $\sqrt{2} \text{Im} H_2^0$.

**The Higgs basis and the alignment limit**

The neutral scalar $H_1^0$ is aligned in field space with the vacuum expectation value $v$. If $\sqrt{2} \text{Re} H_1^0 - v$ were a mass eigenstate, then its tree-level properties would coincide with the Higgs boson of the SM.

In the Higgs basis, the scalar potential is given by:

$$
V = Y_1 H_1^\dagger H_1 + Y_2 H_2^\dagger H_2 + [Y_3 H_1^\dagger H_2 + \text{h.c.}] + \frac{1}{2} Z_1 (H_1^\dagger H_1)^2 \\
+ \frac{1}{2} Z_2 (H_2^\dagger H_2)^2 + Z_3 (H_1^\dagger H_1)(H_2^\dagger H_2) + Z_4 (H_1^\dagger H_2)(H_2^\dagger H_1) \\
+ \left\{ \frac{1}{2} Z_5 (H_1^\dagger H_2)^2 + \left[ Z_6 (H_1^\dagger H_1) + Z_7 (H_2^\dagger H_2) \right] H_1^\dagger H_2 + \text{h.c.} \right\},
$$

After minimizing the scalar potential, $Y_1 = -\frac{1}{2} Z_1 v^2$ and $Y_3 = -\frac{1}{2} Z_6 v^2$. 
For simplicity, assume a CP-conserving scalar potential (where all Higgs basis parameters can be chosen real). The CP-even Higgs squared-mass matrix is,

\[ M_H^2 = \begin{pmatrix} Z_1 v^2 & Z_6 v^2 \\ Z_6 v^2 & m_A^2 + Z_5 v^2 \end{pmatrix}. \]

where \( m_A \) is the mass of the CP-odd Higgs scalar, \( A \).

The CP-even Higgs bosons are \( h \) and \( H \) with \( m_h \leq m_H \). Approximate alignment arises two limiting cases:

1. \( m_A^2 \gg (Z_1 - Z_5)v^2 \). This is the decoupling limit, where \( h \) is SM-like and \( m_h \sim Z_1 v^2 \).

2. \( |Z_6| \ll 1 \). Then, \( h \) is SM-like if \( m_A^2 + (Z_5 - Z_1)v^2 > 0 \). Otherwise, \( H \) is SM-like. This is alignment with or without decoupling, depending on the value of \( m_A \). The boundary between these two regions is fuzzy (\( m_A \sim 500 \text{ GeV} \)).
In particular, the CP-even neutral scalar mass eigenstates are:

\[
\begin{pmatrix}
H \\
\h
\end{pmatrix}
= \begin{pmatrix}
c_{\beta-\alpha} & -s_{\beta-\alpha} \\
\sqrt{2} \text{ Re } H_{0}^{0} - v
s_{\beta-\alpha} & c_{\beta-\alpha}
\end{pmatrix}
\begin{pmatrix}
\sqrt{2} \text{ Re } H_{1}^{0} - v \\
\sqrt{2} \text{ Re } H_{2}^{0}
\end{pmatrix},
\]

where \(c_{\beta-\alpha} \equiv \cos(\beta - \alpha)\) and \(s_{\beta-\alpha} \equiv \sin(\beta - \alpha)\) are defined in terms of the mixing angle \(\alpha\) that diagonalizes the CP-even Higgs squared-mass matrix when expressed in the \(\Phi_{1} – \Phi_{2}\) basis of scalar fields, \(\{\sqrt{2} \text{ Re } \Phi_{1}^{0} - v_{1}, \sqrt{2} \text{ Re } \Phi_{2}^{0} - v_{2}\}\), and \(\tan \beta \equiv v_{2}/v_{1}\).

Since the SM-like Higgs boson must be approximately \(\sqrt{2} \text{ Re } H_{I}^{0} - v\), it follows that

- \(h\) is SM-like if \(|c_{\beta-\alpha}| \ll 1\) (alignment with or without decoupling, depending on the value of \(m_{A}\)),
- \(H\) is SM-like if \(|s_{\beta-\alpha}| \ll 1\) (alignment without decoupling).
The alignment limit in equations

The CP-even Higgs squared-mass matrix yields,

\[ Z_1 v^2 = m_h^2 s_{\beta-\alpha} + m_H^2 c_{\beta-\alpha}, \]

\[ Z_6 v^2 = (m_h^2 - m_H^2) s_{\beta-\alpha} c_{\beta-\alpha}, \]

\[ Z_5 v^2 = m_H^2 s_{\beta-\alpha} + m_h^2 c_{\beta-\alpha} - m_A. \]

If \( h \) is SM-like, then \( m_h^2 \simeq Z_1 v^2 \) (i.e., \( Z_1 \simeq 0.26 \)) and

\[ |c_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(m_H^2 - Z_1 v^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1, \]

If \( H \) is SM-like, then \( m_H^2 \simeq Z_1 v^2 \) (i.e., \( Z_1 \simeq 0.26 \)) and

\[ |s_{\beta-\alpha}| = \frac{|Z_6| v^2}{\sqrt{(m_H^2 - m_h^2)(Z_1 v^2 - m_h^2)}} \simeq \frac{|Z_6| v^2}{m_H^2 - m_h^2} \ll 1. \]
The Higgs–fermion Yukawa couplings

Introduce the following notation: \( U = (u, c, t) \) and \( D = (d, s, b) \) are the mass-eigenstate quark fields, \( K \) is the CKM mixing matrix, \( N = (\nu_e, \nu_\mu, \nu_\tau) \) and \( E = (e, \mu, \tau) \) are the mass-eigenstate lepton fields, and \( \kappa \) and \( \rho \) are \( 3 \times 3 \) Yukawa coupling matrices. In the Higgs basis, the Yukawa Lagrangian is,

\[
-\mathcal{L}_Y = \overline{U}_L (\kappa^U H_1^0 \kappa^U H_1^0 + \rho^U H_2^0 \kappa^U H_1^0 + \rho^U H_2^0) U_R \\
+ \overline{D}_L K \kappa^D \kappa^D \kappa^D \kappa^D H_1^0 H_1^0 + \rho^D \kappa^D \kappa^D \kappa^D \kappa^D H_2^0 H_2^0) D_R \\
+ \overline{N}_L \kappa^E \kappa^E \kappa^E \kappa^E H_1^0 H_1^0 + \rho^E \kappa^E \kappa^E \kappa^E \kappa^E H_2^0 H_2^0) E_R + \text{h.c.},
\]

where \( F_{R,L} \equiv P_{R,L} F \) for \( F = U, D, N \) and \( E \), and \( P_{R,L} \equiv \frac{1}{2} (1 \pm \gamma_5) \) are the right and left-handed projection operators, respectively. By setting \( H_1^0 = v/\sqrt{2} \) and \( H_2^0 = 0 \), one relates \( \kappa^F (F = U, D, E) \) to the \( 3 \times 3 \) diagonal up-type, down-type and charged lepton mass matrices, \( M_F = v \kappa^F / \sqrt{2} \). However, the complex matrices \( \rho^F \) are unconstrained.
In the case of the CP-conserving scalar potential (in the Higgs basis),

\[-\mathcal{L}_Y = \frac{1}{v} \sum_{F=U,D,E} \bar{F} \left\{ s_{\beta-\alpha} M_F + c_{\beta-\alpha} M_F^{1/2} \left[ \rho^F_R + i \varepsilon_F \gamma_5 \rho^F_I \right] M_F^{1/2} \right\} F h \]

\[+ \frac{1}{v} \sum_{F=U,D,E} \bar{F} \left\{ c_{\beta-\alpha} M_F - s_{\beta-\alpha} M_F^{1/2} \left[ \rho^F_R + i \varepsilon_F \gamma_5 \rho^F_I \right] M_F^{1/2} \right\} F H \]

\[+ \frac{1}{v} \sum_{F=U,D,E} \bar{F} \left\{ M_F^{1/2} \left( \rho^F_I - i \varepsilon_F \gamma_5 \rho^F_R \right) M_F^{1/2} \right\} F A \]

\[+ \frac{\sqrt{2}}{v} \left\{ \bar{U} \left[ K M_D^{1/2} \left( \rho^D_R - i \rho^D_I \right) M_D^{1/2} P_R - M_U^{1/2} \left( \rho^U_R - i \rho^U_I \right) M_U^{1/2} K P_L \right] D H^+ \]

\[\bar{N} M_E^{1/2} \left( \rho^E_R - i \rho^E_I \right) M_E^{1/2} P_R E H^+ + \text{h.c.} \right\}, \]

where \( \varepsilon_F = +1 [-1] \) for \( F = U[D,E] \), and

\[\rho^F_R \equiv \frac{v}{2\sqrt{2}} M_F^{-1/2} \left( \rho^F + [\rho^F]^\dagger \right) M_F^{-1/2}, \]

\[\rho^F_I \equiv \frac{-iv}{2\sqrt{2}} M_F^{-1/2} \left( \rho^F - [\rho^F]^\dagger \right) M_F^{-1/2}. \]
In general, the $3 \times 3$ Yukawa matrices, $\rho^F_R$ and $\rho^F_I$ are non-diagonal, which yield flavor non-diagonal couplings of neutral Higgs bosons to the quarks.

Flavor non-diagonal couplings of the neutral Higgs bosons can be naturally eliminated by imposing a symmetry on the 2HDM Lagrangian such that at most one Higgs multiplet is responsible for providing mass for quarks or leptons of a given electric charge [Glashow-Weinberg-Pascos (GWP)]. In the 2HDM, this can be achieved in four different ways.

<table>
<thead>
<tr>
<th>Yukawa coupling models</th>
<th>$\rho^U_R$</th>
<th>$\rho^D_R$</th>
<th>$\rho^E_R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>$1 \cot \beta$</td>
<td>$1 \cot \beta$</td>
<td>$1 \cot \beta$</td>
</tr>
<tr>
<td>Type II (MSSM like)</td>
<td>$1 \cot \beta$</td>
<td>$-1 \tan \beta$</td>
<td>$-1 \tan \beta$</td>
</tr>
<tr>
<td>Type X (lepton specific)</td>
<td>$1 \cot \beta$</td>
<td>$1 \cot \beta$</td>
<td>$-1 \tan \beta$</td>
</tr>
<tr>
<td>Type Y (flipped)</td>
<td>$1 \cot \beta$</td>
<td>$-1 \tan \beta$</td>
<td>$1 \cot \beta$</td>
</tr>
</tbody>
</table>

Four natural choices for the 2HDM Yukawa couplings that yield flavor diagonal neutral Higgs–fermion couplings after imposing the GWP conditions. In all cases, $\rho^F_R$ is proportional to the $3 \times 3$ identity matrix $1$ and $\rho^F_I = 0$. 
What is the meaning of $\tan \beta$?

Prior to imposing the GWP symmetry, $\tan \beta$ and $\alpha$ are not physical parameters of the 2HDM. (All Higgs interactions depend only on the difference, $\beta - \alpha$.)

The GWP mechanism employs a $\mathbb{Z}_2$ symmetry (acting on the dimension-4 terms of the Higgs Lagrangian) that sets $\lambda_6 = \lambda_7 = 0$ in the scalar potential and removes three of the possible six terms (and their hermitian conjugates) of the Yukawa interactions, when expressed in the $\Phi_1$–$\Phi_2$ basis.

<table>
<thead>
<tr>
<th></th>
<th>$\Phi_1$</th>
<th>$\Phi_2$</th>
<th>$U_R$</th>
<th>$D_R$</th>
<th>$E_R$</th>
<th>$U_L, D_L, N_L, E_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type I</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>Type II (MSSM like)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Type X (lepton specific)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>Type Y (flipped)</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>+</td>
<td>-</td>
<td>+</td>
</tr>
</tbody>
</table>

Four possible $\mathbb{Z}_2$ charge assignments that forbid tree-level Higgs-mediated FCNC effects in the 2HDM.

Thus, $\beta$ is the angle that rotates from the Higgs basis to the scalar field basis in which the $\mathbb{Z}_2$ symmetry is manifest.
Even in models of extended Higgs sectors with no (or suppressed) tree-level FCNCs, one can generate neutral flavor changing processes at the loop level.

The most well known example is $b \rightarrow s \gamma$ which is mediated by a virtual charged Higgs boson.

In the Type-II 2HDM, the data for $B \rightarrow X_s + \gamma$ yields a 95% CL limit of:

$$m_{H^\pm} > 580 \text{ GeV}$$

Including additional flavor observables and considering 2HDMs of Types I, II, III=Y and IV=X, exclusions in the $(m_{H^+} - \tan \beta)$ plane have been obtained in A. Arbey, F. Mahmoudi, O. Stal and T. Stefaniak, Eur. Phys. J. C 78, 182 (2018).
Additional flavor constraints arise due to tree-level exchange of a charged Higgs boson.

The largest discrepancy with SM expectations is currently observed in:

\[
\mathcal{R}_D^{SM} = \frac{\mathcal{B}(\bar{B} \to D \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D e^- \bar{\nu}_e)} = 0.300 \pm 0.008 \\
\mathcal{R}_{D^*}^{SM} = \frac{\mathcal{B}(\bar{B} \to D^* \tau^- \bar{\nu}_\tau)}{\mathcal{B}(\bar{B} \to D^* e^- \bar{\nu}_e)} = 0.252 \pm 0.003.
\]

The significance of this deviation is roughly $4\sigma$.

Note: this deviation cannot be accommodated in a Type-II 2HDM.

Figure 6. $\mathcal{R}_D$ and $\mathcal{R}_{D^*}$ measurements: Results from BABAR [28], Belle [34, 35], and LHCb [36], their values and 1-$\sigma$ contours. The average calculated by the Heavy Flavor Averaging Group [37] is compared to SM predictions [19, 20, 21]. ST and HT refer to the measurements with semileptonic and hadronic tags, respectively.

Taken from G. Ciezarek et al., arXiv:1703.01766
Taken from J.P. Lees et al. [BaBar Collaboration], Phys. Rev. D88, 072012 (2013).
Beyond natural suppression of tree-level Higgs-mediated FCNCs

1. Do not assume that the $\rho^F$ are diagonal. Use data to constrain these matrices.

2. Assume flavor alignment in the Higgs sector [A. Pich and P. Tuzon, Phys. Rev. D 80, 091702 (2009)]. That is $\rho^F$ is proportional to $M_F$, or equivalently,

$$\rho^F_R = a^F_R \mathbb{1} \ , \quad \rho^F_I = a^F_I \mathbb{1} ,$$

where $a^F_R$ and $a^F_I$ are real constants.

Note that for Types I, II, X and Y, the constants $a^F_I = 0$ and $a^F_R$ are correlated ($F = U, D, E$). Apart from these special cases, flavor alignment in the Higgs sector is fine-tuned (e.g. not radiatively stable).

3. Assume flavor alignment in the Higgs sector at a very high energy scale (but excluding Types I, II, X and Y). Then, renormalization group (RG) running will generate flavor misalignment at the electroweak scale, which may be small enough to be compatible with experimental constraints.
Summary of the present day constraints and predictions for the heavy Higgs phenomenology, with $\cos(\beta - \alpha) = 0$, $\tan \beta = 10$ and $m_A = m_H = m_{H^\pm} = 400$ GeV. Left panel: Predictions of the leading log approximation. The contours represent the ratio $\text{BR}(H \to b\bar{b}) m_\tau^2 / [\text{BR}(H \to \tau^+ \tau^-) 3 m_b^2]$. The reddish-brown regions are favored by all flavor constraints. Green, blue-gray and tan regions are favored by the measurement of $B \to \tau \nu$, $B_S$ mixing and $B_S \to \mu^+ \mu^-$, respectively. The gray shaded regions produce Landau poles in the Yukawa couplings below $M_P$. Right panel: Result of the parameter scan using full RG running. Blue points correspond to points allowed by the measurement of $B \to \tau \nu$, but not by the measurement of $B_S$ mixing or $B_S \to \mu^+ \mu^-$. Green points are allowed by the measurements of $B \to \tau \nu$ and of meson mixing but not by $B_S \to \mu^+ \mu^-$. Red points are allowed by all constraints. In the solid white region, Landau poles in the Yukawa couplings are produced below the Planck scale. Taken from S. Gori, H.E. Haber and E. Santos, JHEP 1706, 110 (2017).
• In the decoupling limit, $m_h \ll m_H, m_A, m_{H\pm}$. The SM is the effective low energy theory below the mass scale ($Y_2$) of the Higgs basis field $H_2$, and $h$ is the SM-like Higgs boson.

• The inert doublet model (IDM): There is a $\mathbb{Z}_2$ symmetry in the Higgs basis such that $H_2 \rightarrow -H_2$ is the only $\mathbb{Z}_2$-odd field. Then $Z_6 = 0$, and the tree-level properties of $\sqrt{2} \text{Re} H_1 - v$ coincide with the SM Higgs boson. That is, tree-level alignment is exact. Deviations from SM behavior can appear at loop level due to the virtual exchange of the scalar states that reside in $H_2$. The lightest of the $\mathbb{Z}_2$-odd scalars is a dark matter candidate.

• Approximate alignment without decoupling. If present,
  – is this a result of an accidental choice of model parameters?
  – is this a consequence of an approximate (softly-broken) symmetry?
    (The latter is not possible in the IDM.)
Approximate alignment without decoupling in the MSSM

The Higgs sector of the MSSM is a constrained Type-II 2HDM. At tree level, $m_h \leq m_Z$ in conflict with LHC data. However, large radiative corrections can accommodate the observed Higgs mass of 125 GeV (in certain regions of the MSSM parameter space).

The leading effects due to radiative corrections can be exhibited in the limit where $m_h, m_A, m_H, m_{H^\pm} \ll M_{\text{SUSY}},$ where $M_S$ is the SUSY-breaking scale. In this case, one can formally integrate out the squarks and generate a low-energy effective 2HDM Lagrangian that is no longer of the tree-level MSSM form.

taken from M. Carena and H.E. Haber, Prog. Part. Nucl. Phys. 50, 63 (2003).
One must account for the radiative corrections to address the question of Higgs alignment in the MSSM.

The dominant one-loop corrected expressions for $Z_1$ and $Z_6$ are given by$^2$

$$Z_1 v^2 = m_Z^2 c_{2\beta}^2 + \frac{3v^2 s_{\beta}^4 h_t^4}{8\pi^2} \left[ \ln \left( \frac{M_{S}^2}{m_t^2} \right) + \frac{X_t^2}{M_S^2} \left( 1 - \frac{X_t^2}{12M_S^2} \right) \right],$$

$$Z_6 v^2 = -s_{2\beta} \left\{ m_Z^2 c_{2\beta} - \frac{3v^2 s_{\beta}^2 h_t^4}{16\pi^2} \left[ \ln \left( \frac{M_{S}^2}{m_t^2} \right) + \frac{X_t(X_t + Y_t)}{2M_S^2} - \frac{X_t^3 Y_t}{12M_S^4} \right] \right\},$$

where $M_S^2 \equiv M_{SUSY}^2 = m_{\tilde{t}_1} m_{\tilde{t}_2}$, $X_t \equiv A_t - \mu \cot \beta$ and $Y_t = A_t + \mu \tan \beta$.

Note that $m_h^2 \simeq Z_1 v^2$ is consistent with $m_h \simeq 125$ GeV for suitable choices for $M_S$ (as a function of $\tan \beta$ and $X_t$). Exact alignment (i.e., $Z_6 = 0$) is now possible due to an accidental cancellation between tree-level and loop contributions.

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$^2$CP-violating phases that could appear in the MSSM parameters such as $\mu$ and $A_t$ are neglected. The expression for $Z_6$ exhibited above first appears in M. Carena, H.E. Haber, I. Low, N.R. Shah and C.E.M. Wagner, Phys. Rev. D 91, 035003 (2015).
The common wisdom is that the observation of a SM-like Higgs boson at 125 GeV implies that additional Higgs states of the MSSM Higgs sector must be rather heavy (corresponding to the decoupling limit).

Indeed, ATLAS has claimed to rule out $m_A \lesssim 400$ GeV based on Run 1 Higgs data. The corresponding CMS result based on Run 2 Higgs data increases this lower bound to above 500 GeV. But, one needs to be careful about the underlying assumptions...

For example, in the so called MSSM $m_{h_{\text{alt}}}$ benchmark scenario introduced in M. Carena et al., Phys. Rev. D 91, 035003 (2015), the Higgs data places virtually no bound on $m_A$ if $\tan \beta \sim 10$. This is a consequence of the alignment limit without decoupling, which is achieved in the $m_{h_{\text{alt}}}$ benchmark scenario when $\tan \beta \sim 10$. 

Left panel: Regions of the \((m_A, \tan \beta)\) plane excluded in a simplified MSSM model via fits to the measured rates of the production and decays of the SM-like Higgs boson \(h\). Taken from CMS-PAS-HIG-17-031.

Right panel: Likelihood distribution, \(\Delta \chi^2_{\text{HS}}\), obtained from testing the signal rates of \(h\) against a combination of Higgs rate measurements from the Tevatron and LHC experiments, obtained with \texttt{HiggsSignals}, in the alignment benchmark scenario of Carena et al. (op. cit.). From P. Bechtle, S. Heinemeyer, O. Stål, T. Stefaniak and G. Weiglein, Eur. Phys. J. C 75, 421 (2015)
Direct searches for the additional Higgs states also suggest that these states must be heavy, although the sensitivity of these searches are limited if \( \tan \beta \lesssim 6 \).

The observed and expected 95% CL limits on \( \tan \beta \) as a function of \( m_A \) in the MSSM \( m_h^{\text{mod+}} \) benchmark scenario. Left panel: ATLAS results taken from JHEP 01 (2018) 055. Right panel: CMS results taken from arXiv:1803.06553 [hep-ex].
Preferred parameter regions in a pMSSM-8 scan


Points that do not pass the direct constraints from Higgs searches from HiggsBounds and from LHC SUSY particle searches from CheckMATE are shown in gray. Applying a global likelihood analysis to the points that pass the direct constraints, the color code employed is red for $\Delta \chi^2_h < 2.3$, yellow for $\Delta \chi^2_h < 5.99$ and blue otherwise. The best fit point is indicated by a black star.

**Bottom line:** $m_A$ values as low as 200 GeV are still allowed in the MSSM.
How fine-tuned is alignment without decoupling in the MSSM?

Near the alignment limit, \( m_h = 125 \text{ GeV} \) corresponding to \( Z_1 \sim 0.26 \). Parameter regions with \( Z_6 \sim 0.05 \) are compatible with approximate alignment without decoupling (to be compared with \( Z_6 = 0 \) at exact alignment).
Alignment without decoupling due to an approximate symmetry

Consider the following Higgs “family” symmetries that relate the two Higgs doublet fields in the $\Phi_1-\Phi_2$ basis.

<table>
<thead>
<tr>
<th>symmetry</th>
<th>$m_{22}^2$</th>
<th>$m_{12}^2$</th>
<th>$\lambda_2$</th>
<th>$\lambda_4$</th>
<th>$\lambda_5$</th>
<th>$\lambda_6$</th>
<th>$\lambda_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{Z}_2$: $\Phi_1 \to \Phi_1$, $\Phi_2 \to -\Phi_2$</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Pi_2$: $\Phi_1 \leftrightarrow \Phi_2$</td>
<td>$m_{11}^2$</td>
<td>real</td>
<td>$\lambda_1$</td>
<td>real</td>
<td>$\lambda_6^*$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$U(1)$: $\Phi_1 \to e^{-i\theta} \Phi_1$, $\Phi_2 \to e^{-i\theta} \Phi_2$</td>
<td>0</td>
<td></td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\mathbb{Z}_2 \otimes \Pi_2$</td>
<td>$m_{11}^2$</td>
<td>0</td>
<td>$\lambda_1$</td>
<td>real</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$U(1) \otimes \Pi_2$</td>
<td>$m_{11}^2$</td>
<td>0</td>
<td>$\lambda_1$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$SO(3)$: $\Phi_i \to U_{ij} \Phi_j$ [$U \in U(2)/U(1)_Y$]</td>
<td>$m_{11}^2$</td>
<td>0</td>
<td>$\lambda_1$</td>
<td>$\lambda_1 - \lambda_3$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Remark:

The $\Pi_2$ symmetry is equivalent to $\mathbb{Z}_2$ symmetry in a different basis. Otherwise, the above list constitutes a nearly complete classification of inequivalent symmetries of the 2HDM scalar potential.\(^3\)

---

Consider the CP-conserving 2HDM. The scalar potential parameters in the \( \Phi_1-\Phi_2 \) basis are related to the corresponding Higgs basis parameters; e.g.,

\[
Y_3 = \frac{1}{2}(m_{22}^2 - m_{11}^2)s_2\beta - m_{12}^2c_2\beta.
\]

If \( m_{11}^2 = m_{22}^2 \) and \( m_{12}^2 = 0 \), then \( Y_3 = 0 \). The scalar potential minimum condition \( (Y_3 = -\frac{1}{2}Z_6v^2) \) then yields \( Z_6 = 0 \), i.e. exact alignment. This leads to three possible symmetry choices: \( \mathbb{Z}_2 \otimes \Pi_2 \), \( U(1) \otimes \Pi_2 \) or \( SO(3) \). The latter two symmetries will lead to a massless scalar after symmetry breaking. To avoid this, one must include the soft-symmetry-breaking term proportional to a nonzero \( m_{12}^2 \). In these cases, the Higgs alignment will only be approximate.

Unfortunately, none of these symmetries can be extended to the Yukawa interactions without generating a massless quark or some other phenomenologically untenable feature.\(^4\)

Bhupal Dev and Pilaftsis took a different approach. The relation between $Z_6$ and the parameters of the scalar potential in the $\Phi_1-\Phi_2$ basis,

$$Z_6 = -\frac{1}{2} \left[ \lambda_1 c_\beta^2 - \lambda_2 s_\beta^2 - \lambda_{345} c_\beta^2 \right] s_\beta + \lambda_6 c_\beta c_3 \beta + \lambda_7 s_\beta s_3 \beta,$$

where $\lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$, suggests that exact alignment (independently of $\tan \beta$) can be achieved if

$$\lambda_1 = \lambda_2 = \lambda_{345}, \quad \lambda_6 = \lambda_7 = 0,$$

which was called a *natural alignment condition*.\(^5\) Bhupal Dev and Pilaftsis argued for imposing the natural alignment condition at some very high energy scale. Deviations from alignment (due to, e.g., hard breaking from the Yukawa interactions) would be generated by RG-evolution, but might be small enough to be consistent with the Higgs data.

In fact, one can show that for any scalar potential with a $U(1) \otimes \Pi_2$ symmetry in the $\Phi_1-\Phi_2$ basis, there exists another choice of basis in which the natural alignment condition above is satisfied.

The $\mathbb{Z}_2 \otimes \Pi_2$-symmetric 2HDM with mirror fermions

The parameters of the 2HDM scalar potential with a $\mathbb{Z}_2 \otimes \Pi_2$ discrete symmetry satisfy,

$$m_{11}^2 = m_{22}^2, \quad \lambda_1 = \lambda_2, \quad \lambda_5 \text{ real}, \quad m_{12}^2 = \lambda_6 = \lambda_7 = 0.$$ 

Following P. Draper, H.E. Haber and J.T. Ruderman, JHEP 1606, 124 (2016), we extend this symmetry to the Yukawa sector, by introducing mirror fermions. SM fermions are denoted by lower case letters (e.g. left-handed doublet fields $q$ and right-handed singlet fields $u$ and $d$); mirror fermions by upper case letters. Under the discrete symmetries,

$$\Pi_2 : \Phi_1 \leftrightarrow \Phi_2, \quad q \leftrightarrow q, \quad u \leftrightarrow U, \quad \overline{U} \leftrightarrow \overline{U},$$

$$\mathbb{Z}_2 : \Phi_1 \rightarrow \Phi_1, \quad \Phi_2 \rightarrow -\Phi_2, \quad q \rightarrow q, \quad u \rightarrow -u, \quad U \rightarrow U, \quad \overline{U} \rightarrow \overline{U},$$

where $\overline{U}$ is in the representation conjugate to $U$ (to avoid anomalies). The Yukawa couplings consistent with the $\mathbb{Z}_2 \otimes \Pi_2$ discrete symmetry are

$$\mathcal{L}_{\text{Yuk}} \supset y_t (q \Phi_2 u + q \Phi_1 U) + \text{h.c.}$$

\footnote{The down-type fermions and leptons can also be included by introducing the appropriate mirror fermions.}
The model is not phenomenologically viable due to experimental limits on mirror fermion masses (which must be $\gtrsim 1$ TeV). Thus, we introduce a vector-like mass,

$$\mathcal{L}_{\text{mass}} \supset M_U U \bar{U} + \text{h.c.}$$

which preserves the $\mathbb{Z}_2$ but explicitly breaks the $\Pi_2$ discrete symmetry. This symmetry breaking is soft, so that $m_{22}^2 - m_{11}^2$ is protected from quadratic sensitivity to the cutoff scale $\Lambda$.

**Effects of the softly-broken $\Pi_2$ discrete symmetry**

$$\Delta m^2 \equiv m_{22}^2 - m_{11}^2 \sim -\frac{3y_t^2 M_U^2}{4\pi^2} \ln(\Lambda/M_U),$$

neglecting finite threshold corrections proportional to $M_U^2$. Due to the unbroken $\mathbb{Z}_2$ symmetry, a nonzero $m_{12}^2$ is not generated in this approximation.
Integrating out the mirror fermions below the scale $M_U$, one generates a splitting between $\lambda_1$ and $\lambda_2$. Above the scale $M_U$, the diagrams contribute equally to $\lambda_2(\Phi_2^\dagger\Phi_2)^2$ and $\lambda_1(\Phi_1^\dagger\Phi_1)^2$, respectively. Below the scale $M_U$, the diagrams with internal $U$ lines decouple, which then yields

$$\Delta \lambda \equiv |\lambda_1 - \lambda_2| \sim \frac{3y_t^4}{4\pi^2} \log(M_U/m_t) \sim 0.1,$$

for $M_U \sim 1$ TeV. This is a small correction, which in first approximation can be neglected in our analysis.

Note: Nonzero values for $\lambda_6$ and $\lambda_7$ are not generated due to the unbroken $\mathbb{Z}_2$. 
The important parameters of the scalar potential are:

\[ m^2 \equiv \frac{1}{2}(m^2_{11} + m^2_{22}), \quad \Delta m^2 \equiv m^2_{22} - m^2_{11}, \quad R \equiv \frac{\lambda_{345}}{\lambda}, \]

where \( \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5 \). We impose \( \lambda > 0 \) and \( R > -1 \) to ensure that the vacuum is bounded from below. Solving for the potential minimum, there are two possible phases:

- The inert phase, where the \( \mathbb{Z}_2 \) is unbroken, corresponding to the IDM.
- A mixed phase where both \( v_1 \neq 0 \) and \( v_2 \neq 0 \).

In the case of the mixed phase, \( m^2 = -\frac{1}{4}\lambda(1 + R)v^2 \) and

\[ \tan \beta \equiv \frac{v_2}{v_1} = \sqrt{\frac{1 - \epsilon}{1 + \epsilon}}, \quad \text{where} \quad \epsilon = \cos 2\beta = \frac{2\Delta m^2}{\lambda(1 - R)v^2}. \]

The positivity of \( v_1^2 \) and \( v_2^2 \) and the curvature at the extremum requires \( |R| < 1 \) and \( |\epsilon| < 1 \).
**Scalar spectrum of the mixed phase**

The Higgs boson mass spectrum is,

\[
m_{h,H}^2 = \frac{1}{2} \lambda v^2 (1 \mp \sqrt{R^2 + (1 - R^2) \epsilon^2}),
\]

\[
m_A^2 = |\lambda_5| v^2, \quad m_{H^\pm}^2 = m_A^2 - \frac{1}{2}(\lambda_4 - \lambda_5)v^2.
\]

If \( h \) is SM-like, then

\[-1 < R < -\frac{\epsilon^2}{1 - \epsilon^2},\]

and

\[c_{\beta-\alpha} \simeq \frac{\epsilon(1 - R)}{2|R|} + \mathcal{O}(\epsilon^2).\]

In particular, the alignment limit favors small \( |\epsilon| \), which yields \( \tan \beta \sim \mathcal{O}(1) \).

In this parameter regime,

\[m_H^2 \simeq m_h^2 \left( \frac{1 + |R|}{1 - |R|} + \mathcal{O}(\epsilon^2) \right).\]
Fine-tuning of the Higgs squared-mass parameters

In models with additional Higgs bosons with masses of order the electroweak scale, it would appear that the fine-tuning problem of the SM is exacerbated. However, in the $\mathbb{Z}_2 \otimes \Pi_2$-symmetric 2HDM, the symmetry used to impose alignment also ensures that all squared-mass parameters are related (hence, only the one fine-tuning condition of the SM remains).

With the $\mathbb{Z}_2 \otimes \Pi_2$ softly broken, $m^2_{12}$ is generated and is proportional to $M^2_U$ (but only logarithmically sensitive to the cutoff $\Lambda$). If one allows 10% fine tuning and imposes the experimental constraints on mirror fermion masses, regions of $\epsilon$ and $R$ exist consistent with alignment without decoupling.

[P. Draper and H.E. Haber, in progress]
Conclusions

- After the discovery of the Higgs boson, we still do not know if it is unique, or whether additional elementary scalars remain to be discovered.

- Searching for evidence of an extended Higgs sector will be an important activity at the LHC for years to come.

- As the Higgs data becomes more precise, small departures from SM-like behavior may be revealed, thereby providing critical clues to the structure of the Higgs sector and more generally physics beyond the SM.

- Precision flavor physics will continue to provide important constraints on extended Higgs sectors.

- A SM-like Higgs boson most likely implies a decoupling regime for additional Higgs scalars. Nevertheless, approximate alignment without decoupling is still viable.